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REMAINDER THEOREM

Problem types

I. To find the remainder of a given polynomial by actual division method and without division.

II. To prove that a devisor is a factor of the given polynomial. This is done by showing that the remainder **equal to zero**.

III. If there is a remainder the devisor is **not a factor** of the given polynomial expression.

IV. **To find a single unknown constant**, this may be the coefficient of x^3, x^2 or x terms. These problems to find the **single unknown** constant can be further can be subdivided.

a. A single polynomial with a single unknown constant and a factor without remainder or divisor with remainder.

b. Where the **relationship between the two remainders** is given.

i. A single polynomial with two divisors and their respective remainders.

a. When it is a factor the equation obtained after substituting $f(x)$ in the given polynomial is equated to zero.
Equation 1 = 0

b. When the remainder is given, the equation obtained after substituting $f(x)$ in the given polynomial is equated to the remainder.
Equation 2 = remainder.

ii. Two polynomials with a single divisor and their respective remainders. R_1 & R_2
Equation 1 = R_1
Equation 2 = R_2
Solve with linear equations.

c. Where the **relationship between the remainders and the unknown constant is given.**

i. A single polynomial with two given divisors and their respective remainders.

ii. Two polynomials with a single divisor and their respective remainders.

V. **To find two unknown constants using simultaneous equations.**

a. A **single polynomial** with one factor given with no remainder and one divisor given with a respective remainder is given.
Equation 1 (factor) = 0
Equation 2 (divisor) = R (remainder)

b. A **single polynomial** with two factors given.
Equation 1 (factor 1) = 0
Equation 2 (factor 2) = 0

c. A **single polynomial** with two unknown constants, two linear divisors and respective remainders is given.
Equation 1 = Remainder 1
Equation 2 = Remainder 2

d. A **single polynomial** with two unknown constants and two given factors.
Equation 1 (factor 1) = 0
Equation 2 (factor 2) = 0

e. A **single polynomial** with two unknown constants, a quadratic divisor and remainders is given.

f. **Two polynomials** with two unknown constants a single divisor and its respective remainders is given.
Equation 1 (polynomial 1) = Remainder 1
Equation 2 (polynomial 2) = Remainder 2

g. **Two polynomial** with two unknown constants and two linear divisors with their respective remainders is given.
Equation 1 (polynomial 1 & Divisor 1) = Remainder 1
Equation 2 (polynomial 2 & Divisor 2) = Remainder 2

h. **Two polynomials** with two unknown constants and a common factor is given.

VI. **To completely factorise a given polynomial.**

Type A –

a. We may be given a divisor and asked to prove it is a factor and then completely factorise the given polynomial.

Type B -

b. We may be given an unknown single constant and told that a divisor is a factor. We then have to find the unknown constant and then substitute it in the given polynomial to get the complete polynomial. Then we proceed to factorise completely.

Type C -

c. A factor is directly given and we use algebraic division to get a quadratic quotient. The quadratic quotient is further factorised by simple factorisation method to get two more factors.

Type D –

a. By trial and error method (inspection method) we have to find the first factor.

In all the four subtypes (A to D) after finding the first factor, we have to use algebraic division to find a quadratic quotient. This quadratic quotient is further factorised to get two more factors by simple factorisation method.

VII. Finding a number that must be added or subtracted from a given polynomial to make the resulting polynomial completely divisible by a divisor or leaves a certain given remainder.

Solutions to all these problems

1. Write down the given single polynomial or the two given polynomials.
2. Write down equate divisor to zero and find the value of x.
3. Substitute $f(x)$ in the given polynomial.
   i. This may help you to find the remainder.
   ii. This may help you to prove the given divisor is a factor, when the remainder is zero.
   iii. It may help you form a linear equation to find the value of the single unknown constant
   iv. It may help you form to simultaneous equations which must be solved to find the two unknown constants.
4. When we are asked to factorise completely use algebraic division to find the quotient which is a quadratic equation can further be factorised to find two additional factors.
5. When the term says completely factorised it means that all divisors are factors, therefore the remainder is zero. Therefore each factor is equated to zero.
6. In the case of two unknown constants if both the terms for the given polynomial are stated to be factors, then both equations are equated to zero.
   Equation 1 = 0
   Equation 2 = 0
7. If in the case of two unknown constants for a given polynomial if one term is a factor and the other term a divisor with a given remainder, then
   Equation 1 = 0 (in the case of the factor)
   Equation 2 = R (in the case of the remainder). Take R to the other side to create a simultaneous equation.
   Equation 2 – R = 0
8. In the case of two unknown constants where both given terms are divisors with respective remainders.
   Equation 1 = R_1
   Equation 2 = R_2.
   Taking R_1 & R_2 to the LHS we get two simultaneous equations.
9. In the case of a one unknown constant, where we have a single polynomial with two divisors or two polynomials with a single divisor, both giving the same remainder then
   Equation 1 = R
   Equation 2 = R
   Therefore, Equation 1 = Equation 2.
10. **Steps for problems in general**

Step 1 : Write the polynomial identity in descending order.

Step 2 : Equate the divisor to zero and find the value of x which will eliminate any unknown quotient.

Step 3 : Substitute the value of x in the given polynomial

Step 4 : Simplify to find the value.

i. If the remainder is zero the divisor is a factor.
ii. If there is a remainder the divisor is not a factor.
iii. If it is given that the divisor is a factor, equate the value obtained by substituting $f(x)$ in the given polynomial to zero.
iv. If a remainder is given and we are asked to find the unknown constant equate the value obtained by substituting $f(x)$ in the given polynomial to the remainder.

Step 5 : If the remainder is zero then the divisor will be a factor of the given polynomial.

Step 6 : If the divisor leaves the remainder is not a factor for the given polynomial.

Step 7 : Solve linear, simultaneous equations as required.

Key points for solving problems with unknown constant.

(i) Decide whether there are one unknown (linear equation) or two unknown (simultaneous) constants.
(ii) Decide whether there is one or two given polynomial expressions.
(iii) Decide what is given.
a. Two factors
b. one factor and one divisor with remainder.
c. Two divisors with respective remainders. (R_1 & R_2)

Key points for solving problems when a polynomial expression is given but no factors are given.

i. This is known as Trial & Error Method or Inspection Method.
ii. Take $x = 1$, and substitute in the polynomial expression. If the remainder is zero $(x - 1)$ is a factor. $x = 1$, $x - 1 = 0$, $(x - 1)$ is a factor.
iii. If we don't get a remainder zero then try and substitute $x = -1$ in the polynomial expression. if a remainder is zero is obtain then $x = -1$ is a factor. $x + 1 = 0$ is a factor.
iv. If you are unsuccessful with 1 and – 1 to get a remainder of zero then try with $x = 2$, then $(x - 2)$ is a factor and $x = -2$, then $(x + 2)$ is a factor, if the remainder is zero.
v. Similarly try with $x = 3$ and $x = -3$, if unsuccessful with +2 & - 2.
vi. Keep substituting till you get a remainder of zero for a given number.
vii. So the order of trial with numbers is (+1, - 1, +2, - 2, +3, - 3) being substituted in the given polynomial expression to get the remainder of zero.

<u>Problems on Remainder Theorem</u>

1. Find the remainder (without division) when $2x^3 - 3x^2 + 7x - 8$ divided by $(x - 1)$.

Problem type – Single polynomial with single divisor. Ask to find the remainder.

$2x^3 - 3x^2 + 7x - 8$	Step 1: Write the polynomial identity in descending order
$x - 1 = 0$ $x = 1$ $f(x) = f(1)$	Step 2 : Equate the divisor to zero and find the value of x which will eliminate any unknown quotient.
$2(1)^3 - 3(1)^2 + 7(1) - 8$ $f(1) = 2 - 3 + 7 - 8$ $f(1) = 9 - 11 = -2.$	Step 3 : Substitute the value of x in the given polynomial
Remainder $= -2$ ∴ $(x - 1)$ is not factor for polynomial. $2x^3 - 3x^2 + 7x - 8 = 0$	Step 4: If there is a remainder the divisor is not a factor.

Given,

i. (Polynomial) Dividend = $2x^3 - 3x^2 + 7x - 8$
ii. Divisor = $(x - 1)$
iii. Coefficient = $x^3 = 2$
= $x^2 = -3$
= $x = 7$

Constant = − 8
Ask to find the Remainder.

2. $kx^3 - 9x^2 + 4x - 10$ is divided by $(x + 1)$, the remainder is 2. Find the value of constant k.

Problem type – single polynomial expression with single divisor with remainder given. To find the value of a single unknown constant.

$kx^3 - 9x^2 + 4x - 10$	Step (1): Write the polynomial identity in descending order
$x + 1 = 0$ $x = -1$	Step (2): Equate divisor to zero and find the value of x, determine the unknown
$kx^3 - 9x^2 + 4x - 10 = 2$	Step (3): Equate the polynomial to the given remainder.
$k(-1)^3 - 9(-1)^2 + 4(-1) - 10 = 2$ $-k + 9 - 4 - 10 = 2$ $k - 5 = 2$ $-k = 2 + 5$ $-k = 7$ $k = -7$	Step (4): Substitute $f(x)$
$k = -7$	Step (5) A single polynomial with an unknown constant and a divisor with a remainder given.

3. When divided by $x - 3$, the polynomials $x^3 - px^2 + x + 10$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'.

Problem type single polynomial expression with single divisor with given remainder. To find single unknown constant.

$x^3 - px^2 + x + 10$	$2x^3 - x^2 - (p + 3)x - 6$	Step 1 : Write down the given polynomials
$x - 3 = 0$ $x = 3$	$x - 3 = 0$ $x = 3$	Step 2 : Equate the given divisor to zero and obtained the value of x
$(3)^3 - p(3)^2 + 3 + 6$ $27 - 9p + 3 + 6$ $36 - 9p$	$2(3)^3 - 3^2 - (p + 3)3 - 6$ $54 - 9 - 3p - 9 - 6$ $54 - 24 - 3p$	Step 3 ; substitute $f(x) = f(3)$ in the given polynomial.
$36 - 9p = R$ **Equation 1**	$30 - 3p = R$ **Equation 2**	Step 3 : Equate the polynomial to the given same remainder
Equation 1 = Equation 2 (remainder is same) $36 - 9p = 30 - 3p$ $36 - 30 = 9p - 3p$ $6p = 6$ $P = 1$		Eqn. 1 = Eqn. = 2 Solving the linear equation and find the value for p.

4. (i) Show that $(x - 3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence factorise $x^3 - 7x^2 + 15x - 9$.

Problem sub type: A single polynomial is given. A divisor is given that must be proven to be a factor (remainder = 0)
Then we are ask to factorise the polynomial completely.

$x^3 - 7x^2 + 15x - 9$.	Step (1): Write the polynomial identity in descending order.
$(x - 3)$ $x = 3$	Step 2 : Equate the given divisor to zero and obtain the value of x.
$(3)^3 - 7(3)^2 + 15(3) - 9$ $= 27 - 63 + 45 - 9$ $= 72 - 72$ $= 0$	Step 3 : substitute $f(x) = f(3)$ in the given polynomial.
$(x - 3)$ is a factor.	The remainder is zero.
$x^2 - 4x + 3$ $x - 3\ \overline{)x^3 - 7x^2 + 15x - 9}$ $\underline{x^3 - 3x^2}$ Subtract $-4x^2 + 15x$ $\underline{-4x^2 + 12x}$	Algebraic division of polynomial Remainder is zero. $x^2 - 4x + 3$ is the quadratic quotient.

Subtract $3x - 9$ $\underline{3x - 9}$ Subtract 0	
$x^2 - 4x + 3$ $x^2 - 3x - 1x + 3$ $x(x - 3) - 1(x - 3)$ $(x - 1)(x - 3)$	Simple factorisation of quadratic quotient. Therefore $(x - 1)$ & $(x - 3)$ are factors.
$(x - 1)$, $(x - 3)$ & $(x - 3)$	Are factors.

5. Show that $(2x + 7)$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence, factorise the given expression completely using factor theorem.
   Problem sub type: A single polynomial is given. A divisor is given that must be proven to be a factor (remainder = 0) Then we are ask to factorise the polynomial completely.

$2x^3 + 5x^2 - 11x - 14$	Step 1 : Write down the given polynomials in descending order
Divisor given is $2x + 7$ $2x + 7 = 0$ $2x = -7$ $x = \frac{-7}{2}$	Step 2 : Equate the given divisor to zero and obtain the value of x.
$2x^3 + 5x^2 - 11x - 14$ $= 2\left(\frac{-7}{2}\right)^3 + 5\left(\frac{-7}{2}\right)^2 - 11\left(\frac{-7}{2}\right) - 14$ $= 2 \times \frac{343}{84} + 5 \times \frac{49}{4} + \frac{77}{2} - 14$ $= \frac{343}{4} + \frac{245}{4} + \frac{77}{2} - \frac{14}{1}$ $= \frac{343+245+154-56}{4}$ $= \frac{399+399}{4} = \frac{0}{4} = 0$	Step 3 : substitute $f(x) = f\left(\frac{-7}{2}\right)$ in the given polynomial. Yes. $(2x + 7)$ is a factor of the given polynomial because remainder is zero.
$x^2 - x - 2$ $2x + 7\sqrt{2x^3 + 5x^2 - 11x - 14}$ $\underline{2x^3 - 7x^2}$ Subtract $-2x^2 - 11x$ $\underline{-2x^2 + 7x}$ Subtract $-4x - 14$ $\underline{-4x - 14}$ Subtract 0	Step 4 : algebraic division of the given polynomial to get a quadratic quotient, with factor $(2x + 7)$
$x^2 - x - 2$	quadratic quotient.
$x^2 - x - 2$ $= x^2 - 2x + 1x - 2$ $= x(x - 2) + 1(x - 2)$ $= (x + 1), (x - 2)$	Simple factorisation of quadratic quotient to get two factors.
$(x + 1)$, $(x - 2)$ & $(2x + 7)$	Three final factors.

6. Find the value of a if $(x - a)$ is a factor of $x^3 - ax^2 + x + 2$
   Problem sub type: A single polynomial is given. A factor is given. To find the single unknown constant.

$x^3 - ax^2 + x + 2$	Step 1 : Write down the given polynomials in descending order
$x - a = 0$ $x = a$	Step 2 : Equate the given divisor (factor) to zero and obtain the value of x.
$x^3 - ax^2 + x + 2$ $= a^3 - a(a)^2 + a + 2$ $= a^3 - a^3 + a + 2$	Step 3 : Substitute the value of $f(x)$ i.e. a in the given polynomial
$a^3 - a^3 + a + 2 = 0$	Step 4 : Equate polynomial expression to zero as $(x - a)$ is a factor and solve linear equation
$a + 2 = 0$, $a = -2$	Find the value of the single unknown constant.

7. Find the value of k, if $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$. Hence determine whether $(x + 5)$ is also a factor. Factorise completely.

   Problem sub type: A single polynomial is given. A factor is given. To find the single unknown constant. Then we are asked to factorise completely.

$x^3 + 2x^2 - kx + 10$	Step 1 : Write down the given polynomial in descending order
$x - 2 = 0$ $x = 2$	Step 2 : Equate the given divisor to zero and obtain the value of x.
$x^3 + 2x^2 - kx + 10$ $= (2)^3 + 2(2)^2 - k(2) + 10$ $= 8 + 8 - 2k + 10$ $= 26 - 2k$	Step 3 : Substitute the value of $f(x) = 2$ in the given polynomial.
$26 - 2k = 0$ $26 = 2k$ $k = \frac{26}{2} = 13$ $k = 13$	Step 4 : Equate polynomial expression to zero because $(x - 2)$ is a factor and solve linear equation.
$k = 13$	Step 5 : Find the value of the single unknown constant.

$x^3 + 2x^2 - kx + 10$ $x^3 + 2x^2 - 13x + 10$	Step 1 : Substitute value of k in the given polynomial to get complete polynomial expression.
$x + 5 = 0$ $x = -5$	Step 2 : Equate the given divisor $(x + 5)$to zero and obtain the value of x.
$x^3 + 2x^2 - 13x + 10$ $= (-5)^3 + 2(-5)^2 - 13(-5) + 10$ $= -125 + 50 + 65 + 10$ $= -125 + 125$ $= 0$	Step 3 : Substitute the value of $f(x) = -5$ in the given polynomial. The remainder is zero
Remainder = zero.	Step 4: The divisor $(x–5)$ is a factor because remainder is zero

$x^2 + 4x - 5$ $x - 2\,\sqrt{x^3 + 2x^2 - 13x + 10}$ $\underline{x^3 - 2x^2}$ Subtract $-4x^2 - 13x$ $\underline{-4x^2 - 8x}$ Subtract $-5x + 10$ $\underline{-5x + 10}$ Subtract 0	algebraic division of the given polynomial to get a quadratic quotient, with factor $(x - 2)$
$x^2 + 4x - 5$	Quadratic quotient.
$x^2 + 4x - 5$ $= x^2 + 5x - x - 5$ $= x(x + 5) - 1(x + 5)$ $= (x - 1)(x + 5)(x - 2)$	Simple factorisation of quadratic quotient to get two factors.

8. If $(x - 2)$ is a factor of $2x^3 - x^2 - px - 2$
(i) find the value of p, (ii) with this value of p, factorise the above expression completely.
Problem sub type: A single polynomial is given. A factor is given. To find the single unknown constant. Then we are asked to factorise completely.

$2x^3 - x^2 - px - 2$	Step 1 : Write down the given polynomial in descending order
$x - 2 = 0$ $x = 2$	Step 2 : Equate the given divisor to zero and obtain the value of x.
$2x^3 - x^2 - px - 2$ $= 2(2)^3 - (2)^2 - p(2) - 2$ $= 16 - 4 - 2p - 2$ $= 10 - 2p$	Step 3 : Substitute the value of $f(x) = 2$ in the given polynomial.
$10 - 2p = 0$ $10 = 2p$ $p = \frac{10}{2} = 5$ $p = 5$	Step 4 : Equate polynomial expression to zero because $(x - 2)$ is a factor and solve linear equation.
$p = 5$	Value of single unknown constant.

$2x^3-x^2-px-2$ $2x^3-x^2-5x-2$	Step 4 : Substitute the value of unknown constant p in the given polynomial to get complete polynomial expression.
$2x^2$ - $3x$+ 1 x - 2 $\sqrt{2x^3 \; - \; x^2-5x-2}$ $\underline{2x^3-4x^2}$ Subtract $3x^2-5x$ - 2 $\underline{3x^2-\;6x}$ Subtract $x-2$ $\underline{x-2}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor (x - 2)
$2x^2$ - $3x$+ 1	Quadratic quotient.
$2x^2 \;-3x+1$ $= 2x^2 \;-2x-x+1$ $=2x(x-1)-1(x-1)$ $= (2x-1)(x-1)$	Simple factorisation of quadratic quotient to get two factors.
$(2x-1)(x-1)$ *and* $(x-2)$	Final three factors of the given polynomial.

9. Use factor theorem to factorise the polynomial completely.

$x^3-3x^2-10x+24$

Problem sub type: A single polynomial is given. Then we are asked to factorise completely. No factor is given. (Trial & Error method / inspection method)

(i) $x^3-3x^2-10x+24$	Step 1 : Write down the given polynomial in descending order.
x = 1, x = - 1 are not factors.	We get remainders with these numbers when substituted in the given polynomial expression.
Put x = 2 $(2)^3-3(2)^2-10(2)+24$ $= 812-20+24$ = 32 – 32 = 0	Step 2 : Use trial and error method (inspection method) to obtain the factor of the polynomial.

	(x – 2) is a factor as remainder is zero,
x^2 - x - 12 x - 2 $\sqrt{x^3-3x^2-10x+24}$ $\underline{x^3-2x^2}$ Subtract $-x^2-10x+24$ $\underline{-x^2+\;2x}$ Subtract $-12x+24$ $\underline{-12x+24}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor (x - 2)
x^2 - x - 12	Quadratic quotient
(x^2 - x – 12) = x^2 - $4x$ + $3x$ – 12 =$x(x-4)$ + 3 $(x-4)$ $(x+3)(x-4)$	Simple factorisation of quadratic quotient to get two factors..
$(x+3)(x-4)$& $(x-2)$	Final three factors for the given polynomial.

(ii) Use factor theorem to factorise the polynomial completely.

$x^3+x^2-4x-\;4$

(ii) $x^3+x^2-4x-\;4$	Step 1 : Write down the given polynomial in descending order.
Put x = 1 $(1)^3+(1)^2-4(1)-\;4$ = 1 + 1 – 4 – 4 = 2 – 4 – 4 = - 6	Step 2 : Find the factor by using trial and error method (inspection method)
Put x = 2 $(2)^3+(2)^2-4(2)-\;4$ = 8 + 4 – 8 – 4 = 0	Yes. (x – 2) is a factor of the given polynomial as the remainder is zero.
x^2 + $3x$ + 2 x - 2 $\sqrt{x^3-3x^2-10x+24}$ $\underline{x^3-2x^2}$ Subtract $3x^2-4x-4$ $\underline{3x^2-6x\;\;}$	Algebraic division of the given polynomial to get a quadratic quotient, with factor (x - 2)

Subtract $2x - 4$ $\underline{2x - 4}$ Subtract 0	
$x^2 + 3x + 2$	Quadratic quotient
$(x^2 + 3x + 2)$ $x^2 + 2x + x + 2$ $x(x+2) + 1(x+2)$ $\boxed{(x+1)(x+2)}$	Simple factorisation of quadratic quotient to get two factors..
$(x+1)(x+2)$& $(x-2)$	Final three factors for the given polynomial.

10. (i) Use the Remainder Theorem to factorise the following expressions: $2x^3 + x^2 - 13x + 6$.

Problem sub type: A single polynomial is given. Then we are asked to factorise completely. No factor is given. (Trial & Error method / inspection method)

$2x^3 + x^2 - 13x + 6$	Step 1 : Write down the given polynomial in descending order.
Put $x = 2$	Step 2 : Use trial and error (inspection method) to find the factor.
$2x^3 + x^2 - 13x + 6$ $2(2)^3 + (2)^2 - 13(2) + 6$ = 16 + 4 – 26 + 6 = 26 – 26 = 0	Yes. $(x - 2)$ is a factor of the given polynomial as the remainder is zero.
$2x^2 + 5x - 3$ $x - 2\sqrt{2x^3 + \ x^2 - 13x + 6}$ $\underline{2x^3 - 4x^2}$ Subtract $5x^2 - 13x + 6$ $\underline{5x^2 + 10x}$ Subtract $-3x + 6$ $\underline{-3x + 6}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor $(x - 2)$.
$2x^2 + 5x - 3$	Quadratic equation
$2x^2 + 6x - x - 3$ $2x(x+3) - 1(x+3$	Simple factorisation of quadratic quotient to get two factors..

$(2x-1)(x+3)$	
$(2x-1)(x+3)$ & $(x-2)$	Final three factors for the given polynomial.

(ii) Use remainder theorem, factorise completely:
$3x^3 + 2x^2 - 19x + 6$

Problem sub type: A single polynomial is given. Then we are asked to factorise completely. No factor is given. (Trial & Error method / inspection method)

$3x^3 + 2x^2 - 19x + 6$	Step 1 : Write down the given polynomial in descending order
Put $x = 2$ $3(2)^3 + 2(2)^2 - 19(2) + 6$ = 24 + 8 – 38 + 6 = 38 – 38 = 0	Step 2: Use trial and error method (inspection method) to find the factor. Yes $(x - 2)$ is a factor of the given polynomial as the remainder is zero.
$3x^2 + \ 8x - 3$ $x - 2\sqrt{3x^3 + 2x^2 - 19x + 6}$ $\underline{3x^3 - 6x^2}$ Subtract $8x^2 - 19x + 6$ $\underline{8x^2 + 16x}$ Subtract $-3x + 6$ $\underline{-3x + 6}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor $(x - 2)$
$3x^2 + 8x - 3$	Quadratic quotient
$3x^2 + 8x - 3$ $3x^2 + 9x - x - 3$ $3x(x+3) - 1(x+3)$ $(3x-1)(x+3)$	Simple factorisation of quadratic quotient to get two factors.
$(3x-1)(x+3)(x-2)$	Final three factors for the given polynomial

11. Find the values of the constants a and b if $(x-2)$ and $(x+3)$ are both factors of the expression: $x^3+ax^2+bx-12$

Problem sub type: A single polynomial is given. Two factors are given. We are ask to find two unknown constants (simultaneous equation)

$x^3+ax^2+bx-12$	Step 1 : Write down the given polynomial in descending order.
$(x-2)=0$, $x=2$ $(x+3)=0$, $x=-3$	Step 2 : Equate the given divisor to zero and obtain the value of x.
$x^3+ax^2+bx-12$ $(2)^3+a(2)^2+b(2)-12$ $8+4a+2b-12$ $4a+2b-4$	Step 3 : Substitute the value of $f(x)=2$ in the given polynomial .
$4a+2b-4=0$	Equate polynomial expression to zero as $(x-2)$ is a factor.
$4a+2b=4$ $\mathbf{2a+b=2}$ **----- Eqn. 1**	Get simultaneous equation 1
$x^3+ax^2+bx-12$ $(-3)^3+a(-3)^2+b(-3)-12$ $-27+9a-3b-12$ $9a-3b-39$	Substitute the value of $f(x)=-3$ in the given polynomial.
$9a-3b-39=0$	Equate polynomial expression to zero as $(x+3)$ is a factor.
$9a-3b=39$ $\mathbf{3a-b=13}$. .... **Eqn. (2)**	Get simultaneous equation 2
$2a+b=2$ Add $\underline{3a-b=13}$ $5a\quad=15$ $a=\frac{15}{5}$ $a=3$	Step 5 : Solve the simultaneous equations to get the value of the two unknown constants $\boldsymbol{a}$ and b.
$2a+b=2$ $2\times3+b=2$ $6+b=2$ $b=2-6$ $b=-4$	Solve the simultaneous equations to get the value of the two unknown constants a and $\boldsymbol{b}$.

12. Given that $x+2$ and $x+3$ are factors of $2x^3+ax^2+7x-b$.

Problem sub type: A single polynomial is given. Two factors are given. Then we are asked to find the value of the two unknown constants.

$2x^3+ax^2+7x-b$	Step 1 : Write down the given polynomial in descending order.
$(x+2)=0$, $x=-2$ $(x+3)=0$, $x=-3$	Step 2 : Equate the given divisors to zero and obtain the values of x for both divisors.
$2x^3+ax^2+7x-b$ $=2(-2)^3+a(-2)^2+7(-2)-b$ $=-16+4a-14-b$ $=4a-b-30$	Step 3 : Substitute the value of $f(x)=-2$ in the given polynomial.
$4a-b-30=0$	Equate polynomial expression to zero as $(x+2)$ is a factor.
$\mathbf{4a-b=30}$ **------ Eqn. (1)**	Get simultaneous equation 1
$2x^3+ax^2+7x-b$ $=2(-3)^3+a(-3)^2+7(-3)-b$ $=-54+9a-21-b$	Substitute the value of $f(x)=-3$ in the given polynomial
$9a-b-75=0$	Equate polynomial expression to zero as $(x+3)$ is a factor.
$\mathbf{9a-b=75}$ **------ Eqn. (2)**	Get simultaneous equation 2
$4a-b=30$ Subtract $\underline{9a-b=75}$ $5a\quad=45$ $a=\frac{45}{5}=9,$ $a=9$	Step 4 : Solve the simultaneous equation to get the value of unknown constants $\boldsymbol{a}$ and b.
$4a-b=30$ $4\times9-b=30$ $36-b=30$ $b=36-30$ $b=6$	Step 5 : Substitute the value of a to get the value of $\boldsymbol{b}$.

13. If (x - 2) is a factor of the expression $x^3 + ax^2 + bx + 6$, when this expression is divided by (x – 3) it leaves the remainder 3. Find the value of a and b.

Problem sub type: A single polynomial is given. A single factor (x – 2)is given. A single divisor (x – 3) with remainder is given. Then we are asked to find the values of two unknown constants.

$x^3 + ax^2 + bx + 6$	Step 1 : Write down the given polynomial in descending order.
$(x - 2) = 0$, $x = 2$ $(x - 3) = 0$, $x = 3$	Step 2 : Equate the given divisors to zero and obtain the value of x in both cases.
$x^3 + ax^2 + bx + 6$ $= (2)^3 + a(2)^2 + b(2) + 6$ $= 8 + 4a + 2b + 6$ $= 4a + 2b + 14$	Step 3 : Substitute the value of $f(x) = 2$ in the given polynomial.
$4a + 2b + 14 = 0$ $4a + 2b = -14$ $2a + b = -7$	Equate polynomial expression to zero as $(x - 2)$ is a factor.
$\mathbf{2a + b = -7}$ **----- Eqn. 1**	Get simultaneous equation 1
$(3)^3 + a(3)^2 + b(3) + 6$ $= 27 + 9a + 3b + 6$ $= 9a + 3b + 33$	Substitute the value of $f(x) = 3$ in the given polynomial.
$9a + 3b + 33 = 3$ $9a + 3b = -30$	Equate polynomial expression to the remainder 3 as $(x - 3)$ is a divisor and not a factor.
$\mathbf{3a + b = -10}$ **------ Eqn. 2**	Get simultaneous equation 2
$2a + b = -7$ Subtract $\underline{3a + b = -10}$ $-a = 3$ $a = -3$,	Step 4 : Solve the equation to get the value of $\boldsymbol{a}$ and b.
$2a + b = -7$ $2 \times -3 + b = -7$ $-6 + b = -7$ $b = -7 + 6$ $b = -1$	Step 5 : Substitute the value of a to get the value of $\boldsymbol{b}$.

14. Using the Remainder and Factor Theorem, factorise the following polynomial: $x^3 + 10x^2 - 37x + 26$

Problem sub type: A single polynomial is given. Then we are asked to factorise completely. No factor is given. (Trial & Error method / inspection method)

$x^3 + 10x^2 - 37x + 26$	Step 1 : Write down the given polynomial in descending order.
Put $x = 1$ $(1)^3 + 10 - 37 + 26$ $37 – 37 = 0$	Step 2 : Use trial and error method i.e., (inspection method) to find the factor of the polynomial. (x – 1) is a factor as it leaves a remainder of zero.
$x – 1 = 0$, $x = 1$	Step 3 : Equate the given divisor to zero and obtain the value of x.
$x^2 + 11x - 26$ x - 1 $\sqrt{x^3 + 10x^2 - 37x + 26}$ $\underline{2x^3 - 4x^2}$ Subtract $11x^2 - 37x + 26$ $\underline{11x^2 + 11x}$ Subtract $-26x + 26$ $\underline{-26x + 26}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor (x - 1)
$x^2 + 11x - 26$	Quadratic quotient
$x^2 + 11x - 26$ $x^2 + 13x - 2x - 26$ $x(x + 13) - 2(x + 13)$ $(x - 2)(x + 13)$	Simple factorisation of quadratic quotient to get two factors.
$(x - 2)(x + 13)$ & $(x + 1)$	Final three factors of the given polynomial.

15. If $(x+3)$ & $(x-4)$ are factors of $x^3+ax^2-bx+24$, find the value of a and b with these values of a and b factorise the given expression.

Problem sub type: A single polynomial is given. We are given two factors. We are asked to find the value of two unknown constants. Then we are asked to factorise completely.

$x^3+ax^2-bx+24$	Step 1 : Write down the given polynomial in descending order
$x+3=0$, $x=-3$ $x-4=0$, $x=4$	Step 2 : Equate the divisors $(x+3)$ $(x-4)$to zero to get x values in both cases.
$(-3)^3+a(-3)^2-b(-3)+24$ $=-27+9a+3b+24=0$ $=9a+3b-3$	Step 3 : substitute *f (x)*= - 3 in the given polynomial.
$9a+3b-3=0$ $3a+b-1=0$	Equate polynomial expression to zero as $(x+3)$ is a factor.
$\mathbf{3a+b=1}$ **------ Eqn. 1**	Get simultaneous equation 1
$(4)^3+a(4)^2-b(4)+24$ $=64+16a-4b+24$ $=16a-4b+88$	Step 3 : substitute *f (x)*= 4 in the given polynomial.
$16a-4b+88=0$ $4a-b+22=0$	Equate polynomial expression to zero as $(x-4)$ is a factor.
$\mathbf{4a-b=-22}$ **--------- Eqn. 2**	Get simultaneous equation 2
$3a+b=1$ Add $\underline{4a-b=-22}$ $7a \quad =-21$ $a=\frac{21}{7}$ $a=-3$	Step 4 : Solve simultaneous equation (1) & (2) to get the value of unknown constants $\boldsymbol{a}$ and b .
$3a+b=1$ $3(-3)+b=1$ $-9+b=1$ $b=1+9$ $b=10$	Step 5 : Substitute the value of a and $\boldsymbol{b}$. In the polynomial and factorise.

x^2-6x+8 $x+3\sqrt{x^3-3x^2-10x+24}$ $\underline{x^3+3x^2}$ Subtract $-6x^2-10x$ $\underline{-6x^2-18x}$ Subtract $8x+24$ $\underline{8x+24}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor $(x+3)$ You can also use the other factor $(x-4)$ for algebraic division.
x^2-6x+8	Quadratic quotient
x^2-6x+8 $=x^2-4x-2x+8$ $=x(x-4)-2(x-4)$ $(x-2)(x-4)$	Simple factorisation of quadratic quotient to get two factors.
$(x-2)(x-4)$ & $(x+3)$	Final three factors of the given polynomial.

16. Given that $(x+1)$ and $(x-2)$ are factors of x^3+ax^2-bx-6, find the value of a and b and factorise the given expression completely.

Problem sub type: A single polynomial identity is given. Two factors are given. We are asked to find two unknown constants. Then we are asked to factorise completely.

x^3+ax^2-bx-6	Step 1 : Write down the given polynomial in descending order
$x+1=0$, $x=-1$ $x-2=0, x=2$	Step 2 : Equate the divisors $(x+1)$ $(x-2)$to zero to get both values x.
x^3+ax^2-bx-6 $=(-1)^3+a(-1)^2-b(-1)-6\ 0$ $=-1+a+b-6$ $=a+b-7$	Step 3 : First substitute *F (x)* $=-1$ in the given polynomial.
$a+b-7=0$	Equate polynomial expression to zero as $(x+1)$ is a factor.
$\boldsymbol{a+b=7}$ **------ Eqn. 1**	Get simultaneous equation 1

x^3+ax^2-bx-6 $=(2)^3+a(2)^2-b(2)-6$ $=8+4a-2b-6$ $=4a-2b+2$	Step 4: Substitute the value of $f(x)=2$ in the given polynomial.
$=4a-2b+2=0$ $4a-2b=-2$ $2a-b=-1$	Equate polynomial expression to zero as $(x-2)$ is a factor.
$2a-b=-1$ ---- Eqn. 2	Get simultaneous equation 2`
$a+b=7$ Add $\underline{2a-b=-1}$ $3a\quad=6$ $a=\frac{6}{3}$ $a=2$	Step 5 : solve the equations to get the value of ***a*** and *b*.
$a+b=7$ $2+b=7$ $b=7-2$ $b=5$	Step 6 : Substitute the value of *a* in any equation to get the value of ***b***
a ↓ b ↓ x^3+2x^2-5x-6	Step 7 : Substitute the value of *a* and *b* in the given polynomial and factorise completely. Quadratic quotient is $(x^2+\ x-6)$ Factorise again to get further factors.
$x^2+\ x-6$ $x+1\,\sqrt{x^3+2x^2-5x-6}$ $\underline{x^3+\ x^2}$ Subtract x^2-5x $\underline{x^2+\ \ x}$ Subtract $-6x-6$ $\underline{-6x-6}$ Subtract 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor $(x+1)$
$x^2+\ x-6$	Quadratic quotient
$x^2+\ x-6$	Simple factorisation of

$x^2+\ 3x-2x-6$ $x(x+3)-2(x+3)$ $(x-2)(x+3)$	quadratic quotient to get two factors.
$(x-2)(x+3)$ & $(x+1)$	Final three factors for the given polynomial.

17. If $(2x+1)$ is a factor of both the expressions $2x^2-5x+p$ and $2x^2+5x+q$, find the value of p and q. Hence find the other factors of both polynomial.

Problem sub type: Two polynomials are given. A single common factor is given. We are asked to find the value of two unknown constants. Then we are asked to factorise completely.

$2x^2-5x+p$ --- (Polynomial 1) $2x^2+5x+q$ ---- (Polynomial 2)	Step 1 : Write down the given polynomials in descending order
$2x+1=0,$ $2x=-1=0,$ $x=(\frac{-1}{2})$	Step 2 : Equate the divisor to zero to get the value of x.
$2x^2-5x+p$ $2(\frac{-1}{2})^2-5\left(\frac{-1}{2}\right)+p$ $2\left(\frac{1}{4}\right)+\frac{5}{2}+p$ $\frac{1}{2}+\frac{5}{2}+p$	Step 3 : First substitute $x=\frac{-1}{2}$ in the first polynomial.
$\frac{1}{2}+\frac{5}{2}+p=0$ $\frac{6}{2}+\ p=\ 0$ $p=-3.$	Equate polynomial expression to zero as $(x-2)$ is a factor.
$p=-3.$	Solve for value of unknown constant.
$2x^2+5x+q$ $=2(\frac{-1}{2})^2+5\left(\frac{-1}{2}\right)+q$ $=2\left(\frac{1}{4}\right)-\frac{5}{2}+q$	First substitute $f(x)=\frac{-1}{2}$ in the second polynomial.

$= \frac{1}{2} - \frac{5}{2} + q$ $\frac{-4}{2} + q$ $q = \frac{4}{2}$ $q = 2$	
$\frac{-4}{2} + q = 0$	Equate polynomial expression to zero as $(2x + 1)$ is a factor.
$q = \frac{4}{2}$ $q = 2$	Solve for the value of the Second unknown constant
Substitute value of p in polynomial 1 and factorise to get 2 factors.	Substitute value of q in polynomial 2 and factorise to get 2 factors.
$2x^2 - 5x + p$ $2x^2 - 5x - 3$ $2x^2 - 6x + x - 3$ $2x(x - 3) + 1(x - 3)$ $(2x + 1)(x - 3)$	$2x^2 + 5x + q$ $2x^2 + 5x + 2$ $2x^2 + 4x + x + 2$ $2x(x + 2) + 1(x + 2)$ $(2x + 1)(x + 2)$

18. If $2x^3 + ax^2 - 11x + b$ leaves remainders 0 and 42 when divided by $(x - 2)$ and $(x - 3)$ respectively, find the values of a and b. With these values of a and b, factorise the given expression.

Problem sub type: A single polynomial is given. Two divisors are given. Their respective remainders are given. Then we are asked to factorise completely.

$2x^3 + ax^2 - 11x + b$	Step 1 : Write down the given polynomial in descending order
$(x - 2) = 0,\ x = 2$ $(x - 3) = 0,\ x = 3$	Step 2 : Equate the two divisors to zero to get both values of x.
$2(2)^3 + a(2)^2 - 11(2) + b$ $16 + 4a - 22 + b$ $4a + b - 6$	Step 3 : Substitute $f(x) = 2$ in given polynomial
$4a + b - 6 = 0$	Equate polynomial expression to remainder zero as $(x - 2)$ is a factor.
$4a + b = 6$ ------- Eqn. (1)	Get simultaneous equation 1
$2x^3 + ax^2 - 11x + b$ $= 2(3)^3 + a(3)^2 - 11(3) + b$ $= 54 + 9a - 33 + b$ $= 9a + b + 21$	Step 3 :Substitute the value of $f(x) = 3$ in the given polynomial
$9a + b + 21 = 42$ $9a + b = 21$	Equate polynomial expression to remainder 42 as $(x - 2)$ is a divisor (not a factor).
$9a + b = 21$ ------- Eqn. 2	Get simultaneous equation 2
$4a + b = 6$ $\underline{9a + b = 21}$ Subtract $-5a = -15$ $a = 3$ $4 \times 3 + b = 6$ $b = 6 - 12$ $b = -12$	Step 3 : Get the equation 1 and 2 and solve them to get the value of a and b.

19. If $2x^3 + ax^2 + bx - 2$ has a factor $(x + 2)$ and leaves the remainder 7 when divided by $(2x - 3)$. Find the value of a and b with these values of a and b factorise the given polynomial completely.

Problem sub type: A single polynomial is given. We are given one factor. We are given a divisor with remainder. Then we are asked to find two unknown constants. Then we are asked to factorise completely.

$2x^3 + ax^2 + bx - 2$	Step 1 : Write down the given polynomial in descending order
$x + 2 = 0,\ x = -2$ $2x - 3 = 0, 2x = 3, x = \frac{3}{2}$	Step 2 : Equate the two divisors to zero to get the value of x.
$2x^3 + ax^2 + bx - 2$ $=2(-2)^3 + a(-2)^2 + b(-2) - 2$ $=-16 + 4a - 2b - 2$	Step 3 : Substitute $f(x) = -2$ in the given polynomial

$4a-2b-18=0$ $4a-2b=18$	Equate polynomial expression to zero as $(x+2)$ is a factor.
$\mathbf{2a-b=9}$ **-------- Eqn. (1)**	Get simultaneous equation 1
$=2(\frac{3}{2})^3+a(\frac{3}{2})^2+b\left(\frac{3}{2}\right)-2$ $=2\times\frac{27}{4}+\frac{9a}{4}+\frac{3b}{2}-2$ $=\frac{27}{4}+\frac{9a}{4}+\frac{3b}{2}-2$ $=\frac{27+9a+6b-8}{4}$	Substitute $f(x)=\frac{3}{2}$ in the given polynomial
$\frac{27+9a+6b-8}{4}=7$ $9a+6b+19=28$ $9a+6b=28-19$ $9a+6b=9$	Equate polynomial expression to remainder 7 as $(2x-3)$ is a divisor (not a factor).
$\mathbf{3a+2b=3}$ **---------Eqn. (2)**	Get simultaneous equation 2
$2a-b=9\times 2$ $3a+2b=3\ \times 1$ $4a+2b=3$ Add : $\underline{3a+2b=3}$ $7a\quad=21$ $a=\frac{21}{7}=3$ $a=3.$	Step 5 : Solve the equation (1) and (2) to get the values of ***a*** and *b*.
$2a-b=9$ $2\times 3-b=9$ $6-b=9$ $6-9=b$ $b=-3$	Step 6 : Substitute the value of *a* to get the value of ***b*** in any one equation.
$2x^3+3x^2-3x-2=0$ ↓ ↓ *a* *b*	Step 7 : Substitute the values of unknown constants *a* and *b* in the polynomial and factorise to get quadratic quotient.

$2x^2-x-1$ $x+2\ \sqrt{2x^3+3x^2-3x-2}$ $\underline{2x^3+4x^2}$ Subtract $-x^2-3x$ $\underline{-x^2-2x}$ Subtract $-x-2$ $\underline{-x-2}$ 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor $(x-1)$
$2x^2-x-1$	Quadratic quotient
$(2x^2-x-1)$ $(2x^2-x-1)$ $(2x^2-2x+x-1)$ $2x(x-1)+1(x-1)$ $(2x+1)(x-1)$	Simple factorisation of quadratic quotient to get two factors.
$(2x+1)(x-1)$ & $(x+2)$	Final factors.

20. Give $f(x)=ax^2+bx+2$ and $g(x)=bx^2+ax+1$, if $(x-2)$ is a factor of $f(x)$ but leaves the remainder -15 when it divides $g(x)$. Find the value of *a* and *b*, factorise the expression $f(x)+g(x)+4x^2+7x$.

Problem sub type: We are given two polynomial expressions. We are given a single divisor which is a factor for the first polynomial but is a divisor with a remainder for the second polynomial. Then we are asked to find the value of both unknown constants. Then we are asked to factorise completely.

$f(x)\ \ ax^2+bx+2$	Step 1 : Write down the given polynomial in descending order
$x-2=0,\ x=2$	Step 2 : Equate the single divisor to zero to get the value of *x*.
ax^2+bx+2 $a(2)^2+b(2)+2$ $=4a+2b+2$	Step 3 :Substitute the value of $f(x)=2$ in the first polynomial expression.
$4a+2b+2=0$	Equate polynomial

	expression to zero as $(x-2)$ is a factor for the first polynomial expression.
$\mathbf{4a+2b=-2}$ **------ Eqn. 1**	Get simultaneous equation 1
bx^2+ax+1 $b(2)^2+2a+1$ 4b + 2a + 1	Substitute the value of *g (x)* = 2 in the second polynomial expression.
4b + 2a + 1 = -15 4b + 2a = - 16 2b + a = - 8	
a **+ 2b = - 8 ----- Eqn. 2**	
$4a+2b=-2$ $a+2b=-8$ $3a \quad = 6$ $a = 2$	Step 5 : Solve the equations (1) and (2) to get the values of *a* and *b*.
$8+2b=-2$ $2b=-10$ $b=-5$	Step 6 : Substitute the value of *b* in any one of the equation to get *a*.
$f(x)\ ax^2+bx+2$ $=(\mathbf{2x^2-5x+2})$ $g(x)=bx^2+ax+1$ $=(\mathbf{-5x^2+2x+1})$	Step 7 : Write down *f (x)* & *g (x)* By substituting *a* and *b* values.
$f(x)+ \quad g(x)+4x^2+7x$ ↓ ↓ $(2x^2-5x+2)+(-5x^2+2x+1)+4x^2+7x$ $=2x^2-5x+2-5x^2+2x+1+4x^2+7x$ $=6x^2-5x^2+9x-5x+2+1$ $=x^2+4x+3$	Step 8: Adding to get the expression $f(x)+g(x)+4x^2+7x$
$\boxed{x^2+4x+3}$ ↓ $x^2+3x+x+3$ $x(x+3)+1(x+3)$ $(x+1)(x+3)$	Step 9 : Simple factorisation of additive term.
$(x+1)(x+3)$& $(x-2)$	Final factors.

21. If $(x - 2)$ is a factor of the expression $2x^3+ax^2+bx-14$, and when the expression is divided by $(x - 3)$ it leaves the remainder 52. Find the value of *a* and *b*.

Problem sub type: A single polynomial is given. We are given one factor and one divisor with remainder. Then we are asked to find the value of both unknown constants. Then we are asked to factorise completely.

$2x^3+ax^2+bx-14$	Step 1 : Write down the given polynomial in descending order
$(x-2)=0$, $x=2$ $(x-3)=0$, $x=3$	Step 2 : Equate the divisors to zero to get the value of *x*.
$2(2)^3+a(2)^2+b(2)-14$ $=16+4a+2b-14$ $=4a+2b+2$	Step 3: Substitute the value of *f (x)* = 2 in the given polynomial expression.
$4a+2b+2=0$	Equate polynomial expression to zero as $(x-2)$ is a factor
$\mathbf{4a+2b=-2}$ **----- Eqn. 1**	Get simultaneous equation 1
$2x^3+ax^2+bx-14$ $(x-3)=0$, $x=3$ $2(3)^3+a(3)^2+b(3)-14$ $54+9a+3b-14$ $9a+3b+40$	Substitute the value of *f (x)* = 3 in the given polynomial expression.
$9a+3b+40=52$ $9a+3b=52-40$ $9a+3b=12\ \ (\div 3)$	Equate polynomial expression to remainder 52 as $(x-3)$ is a divisor (not a factor).
$\mathbf{3a+b=4}$**------ Eqn. 2**	Get simultaneous equation 2
$4a+2b=-2\times 1$ $3a+\ b=\ 4\times 2$ $4a+2b=-2$ Subtract $6a+2b=\ 8$ $-2a \quad =-10$ $2a=10,$	Step 5: Solve the equations (1) and (2) to get the value of *a* and *b*.

$a = \frac{10}{2}$ $a = 5$	
$4a + 2b = -2$ $4 \times 5 + 2b = -2$ $20 + 2b = -2$ $2b = -2 - 20$ $2b = -22$ $b = -11$	Step 6: Substitute the value of a in any one equation to get the value of b.
$2x^3 + ax^2 + bx - 14$ $= 2x^3 + (5)x^2 + (-11)x - 14$ $= 2x^3 + 5x^2 - 11x - 14$	Step 6 : Substitute the value of b in the given polynomial expression.
$2x^2 + 9x + 7$ $x - 2\sqrt{2x^3 + 5x^2 - 11x - 14}$ $\underline{2x^3 - 4x^2}$ Subtract $9x^2 - 11x$ $\underline{9x^2 - 18x}$ Subtract $7x - 14$ $\underline{7x - 14}$ 0	Algebraic division of the given polynomial to get a quadratic quotient, with factor (x - 2)
$2x^2 + 9x + 7$	Quadratic quotient
$2x^2 + 9x + 7$ $= 2x^2 + 2x + 7x + 7$ $= 2x(x + 1) + 7(x + 1)$ $= (2x + 1)(x + 1)$	Simple factorisation of quadratic quotient to get two factors.
$(2x + 1)(x + 1)$ & $(x - 2)$	Final factors of given polynomial

22. Show that $(x - 1)$ if a factor of $x^3 - 7x^2 + 14x - 8$. Hence completely factorise the above expression.
23. Find the value of the constants a and b, if $(x - 2)$ and $(x + 3)$ are both factors of the expression $x^3 + ax^2 + bx - 12$.
24. Find the remainders when $2x^3 - 3x^2 + 7x - 8$ is divided by $x - 1$.
25. Using factor theorem, show that $(x - 3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence, factorise the given expression completely.
26. Find the value of a, if $(x - a)$ is a factor of $x^3 - a^2x + x + 2$.